Knudsen Layer Reduction of Fusion Reactivity

Kim Molvig and Nelson M. Hoffman

(The work is a collaboration with Eric M. Nelson, Brian J. Albright, Evan Dodd, George Zimmerman, and Ed Williams. The idea of Knudsen losses of fuel ions was first suggested by Robert B. Webster)

LA-UR-12-26413

Los Alamos National Laboratory, Los Alamos, NM 87506

Introduction

Over-prediction of fusion energy yield has been a persistent enigma in inertial fusion systems for over 50 years. This paper presents a new piece of physics to help to resolve this puzzle. Our best (clean) theory and code predictions have systematically calculated higher energy yield from fusion than observed in experiments by factors of two and more. This yield over-prediction has traditionally been attributed to the “mix” of impurities into the fuel. Many models of such mix have been developed, refined and applied over the years. Nonetheless, current mix models fall short of being fully satisfactory. This work is motivated by the belief that it is missing physics beyond “mix” that is needed to account for yield over-prediction (although we certainly expect mix to play a role and have combined it with the new physics in simulating experiment). The Knudsen layer effect we describe briefly here (and published in *Physical Review Letters* [1]) is such a piece of physics that fundamentally reduces the fusion reactivity by depleting the high energy tail population of fuel ions that are primarily responsible for the yield.

Basic Theory of Fusion Reactivity and the Knudsen Layer

Nuclear fusion reactions at energies below several hundred keV involve quantum tunneling through the repulsive Coulomb barrier that keeps nuclei from interacting. The probability of successful quantum tunneling rises rapidly as the energy of nuclei increase, which makes the fusion cross section (i.e., the probability of fusion) a rapidly rising function of energy as well (red curve in Fig. 1). In the calculation for fusion reactivity, the rapid increase of cross section competes with the rapid decrease of the number of ions present at a given energy (the ion distribution function, called a “Maxwellian” distribution” in thermal equilibrium; blue curve in Fig. 1),. This competition results in the “Gamow peak” in fusion reactivity (black curve in Fig. 1), which shows where most of the fusion reactions occur. The Gamow peak energy is well above the average energy of the thermal ions.

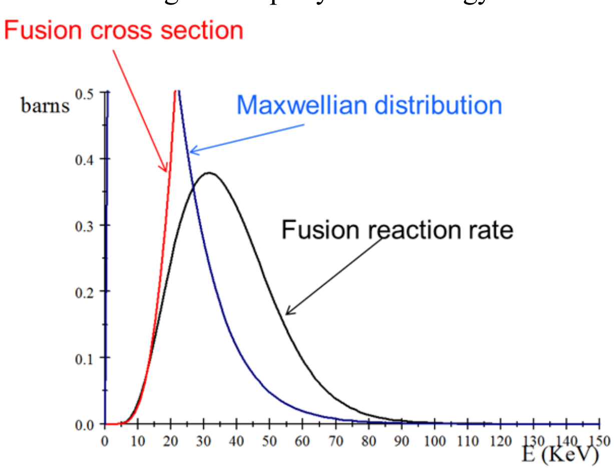

Figure 1: Competing energy dependences of Maxwellian ion distribution and fusion cross section determine Gamow peak in the fusion reactivity.

Inertially confined fusion systems typically have plasma fuel enveloped by a cold non-reacting region or "wall." Typically, thermal ions have mean free paths for Coulomb scattering that are short compared to the distance to this wall. But owing to the rapid decrease of collision probability as ion energy increases, the mean free path for the fusing ions at the Gamow peak can be 10 to 50 times longer. These ions can reach the wall before undergoing a thermalizing collision and therefore be lost to the system, resulting in a depletion of the high energy tail ions that account for most of the fusion reactivity. In practical terms the way this works is shown in Figure 2, where the boundary of a spherical fuel volume is shown as a blue circle. A reference display point at 80% of the fuel radius is indicated. Half the fuel volume lies closer to the wall than this point. The circles surrounding the point indicate the mean free path for thermal ions (red circle) and Gamow peak energy ions (green circle). The case for 2 keV ion temperature (left) requires the zoomed-in image to even see the radii, while at 10 keV temperature (right), although the thermal ions are still not reaching the wall, a high fraction of the the Gamow peak ions do. This picture illustrates how Knudsen layer effect operates in practice. At ignition temperatures (as low as 2 keV in some systems) no effect will be seen. But as fusion proceeds to the higher temperatures of propagating burn (10 keV and above), one can expect efficient escape of fusing ions, resulting in significant reduction of fusion reactivity and therefore a reduction in fusion yield.

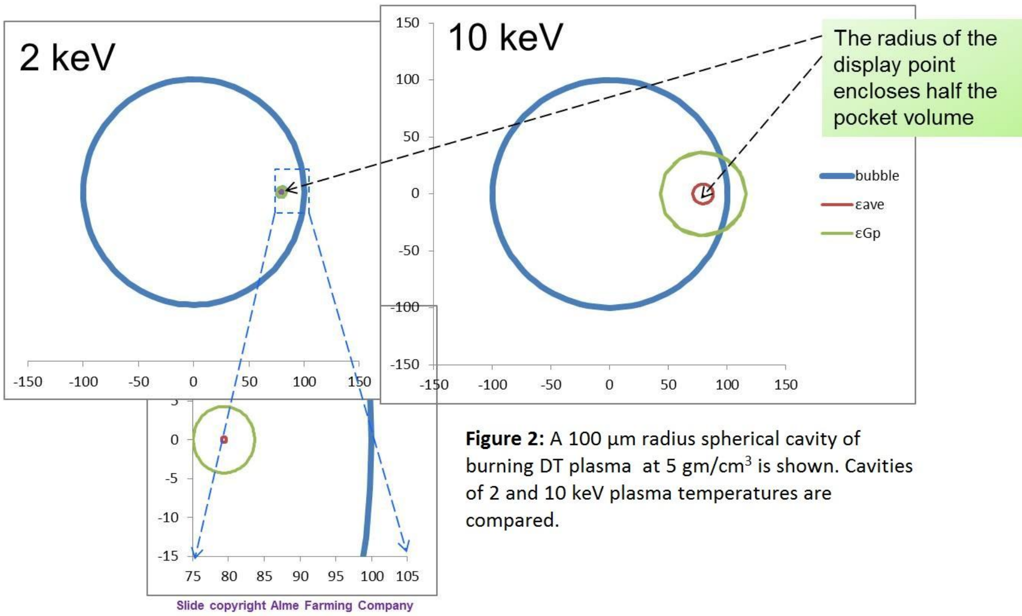

Figure 2: A 100 μm radius spherical cavity of burning DT plasma at 5 gm/cm^3 is shown. Cavities of 2 and 10 keV plasma temperatures are compared.

We have developed a simple theory of the Knudsen tail effect by employing a *kinetic* spatial diffusion model, wherein the diffusion coefficient depended on particle energy and the diffusion operator was replaced by a local loss term of the same magnitude. We could then obtain a kinetic equation for the tail ion distribution function, and an approximate WKB solution for a modified ion distribution function that no longer has a Maxwellian shape. Thus tail depletion depends on the so-called Knudsen number, which is the ratio of thermal mean free path to distance from the wall. From the modified distribution function one sees that while the Knudsen number may be small, strong energy dependence of the modified distribution can make ion losses significant for energies around the Gamow peak. From the modified distribution a fusion reactivity can be computed that is now a function of Knudsen number. The result is that we obtain a non-local reactivity that depends on distance from the wall as well as local ion temperature.

Simulations of ICF capsule implosion experiments on OMEGA

We implemented this simple model in a radiation-hydrodynamics code and performed simulations of implosions of DT-filled ICF capsules conducted at the OMEGA laser [2] during 2005-2011. The implosions were selected to be highly diverse, spanning 2 1/2 orders of magnitude in observed yield and a factor of four in observed ion temperature. Fuel gas molar composition ranged from T:D = 0.55:1 to T:D = 585:1, more than three orders of magnitude. Several of the capsules were the deuterated-shell implosions and corresponding "reference" implosions analyzed by Wilson et al. [3]; these capsules give unambiguous evidence for ion-species mixing at the scale of an ion mean free path (“atomic mix”). Since it is clear that the deuterated shell yield requires mix we used both the Dimonte [4] mix model already in the code and the new combined Knudsen/mix model that contained both.

The model contained two parameters, the mix initial scale length and the fraction of absorbed laser energy, that were varied to best fit the 11 shots simulated. The Knudsen model was used without adjustable parameters. We used a normalized error metric to measure the deviation of simulation from experiment as determined by the three measured quantities: ion temperature, DT neutron yield, and bang time (time of peak neutron production rate). For each model, several hundred 1D simulations were run for each of the eleven capsules in Figure 3, spanning a grid varying the two input parameters: mix scale length and fractional laser source. We then identified the values of the input parameters giving the smallest value of the error metric, averaged over all of the capsules in Figure 3. The result was that for the Knudsen/mix model, the error metric had a minimum value of about 2 standard deviations for a mix scale length of 0.15 µm, while for the mix model alone, the error metric had a minimum value of about 4 standard deviations at a mix scale length of 0.4 µm. In other words, the Knudsen/mix model explained the observations about two times better than the mix model alone.

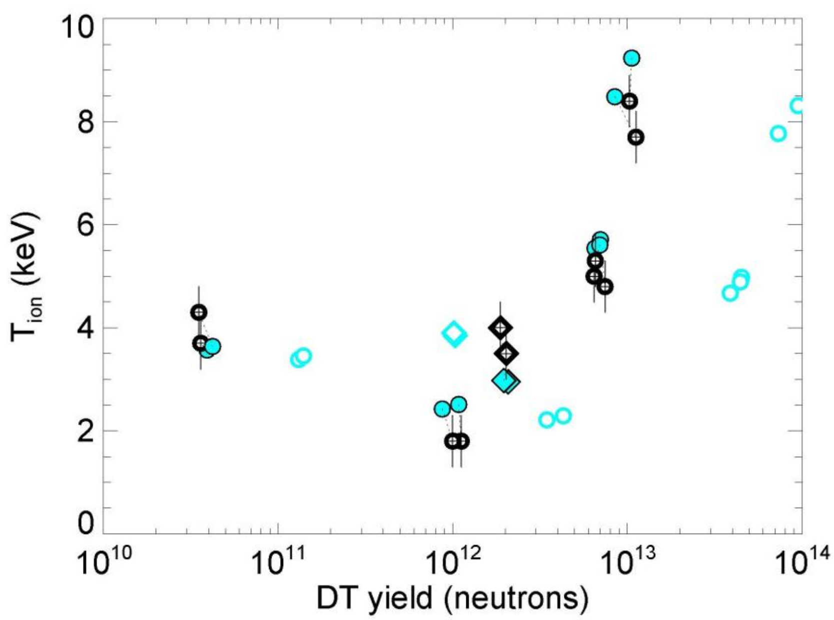

Figure 3. Comparison of observed (black symbols) and simulated (blue symbols) DT neutron yields and ion temperatures for the OMEGA capsules discussed in the text. Open blue symbols show “nominal” simulations. Filled blue symbols show simulations using Knudsen/mix model. Circles indicate ordinary plastic-shell capsules. Diamonds show deuterated-shell capsules. Knudsen/mix model gives markedly improved agreement with observations, compared to nominal.

For the hotter implosions, most of the observed yield reduction is accounted for by the Knudsen model. For cooler implosions, most of the yield reduction comes from mix, and the yield *increase* seen for the deuterated shells also comes from mix. Our Knudsen/mix model handles all of this variation automatically and naturally. The improved agreement between observations and simulations given by the Knudsen/mix model, compared to the mix model alone, is significant, indicating the greater explanatory and predictive power of the Knudsen/mix model.

In summary, we believe the Knudsen layer effect on reactivity to be an important new piece of physics in fusion burn that helps resolve the long standing enigma of over-prediction of yield. Much work needs be done to compute the effect accurately in complex geometry and with a much more detailed treatment of the “wall” interaction with the high energy fuel ions, and this work is currently in progress.

References

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